A Particle Swarm Optimization Algorithm with Novel Expected Fitness Evaluation for Robust Optimization Problems

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Abstract — In this paper, an improved particle swarm optimization (PSO) algorithm for robust optimization problems is proposed. The new algorithm deeps basic concepts of the PSO, results in dynamic determination of the robust optimal solution by using the proposed four-quadrant-longest-distance expected fitness evaluation method in the 2D space, and shows a faster convergence speed and a higher solution accuracy. The efficiency and advantages of the proposed method is verified by numerical experiments.

I. INTRODUCTION

In 1995, an stochastic optimization strategy named particle swarm optimization (PSO) was originally proposed by Dr. Eberhart and Dr. Kennedy inspired by the social behavior associated with swarm of bees [1]. The underlying mechanism of PSO is that, particles move through the problem space influenced by the optimum experience of individual (*pbest*) and swarm (*gbest*) simultaneously [2]. PSO has been proved to be a successful tool used to solve many difficult optimization problems [3], [4]. Traditionally, the ultimate attempt of PSO or other optimization study is to find one global one or several local optimal solution(s) [3]-[5]. However, in the view of robust optimization design technique which is a new technique about ten years old, the "optimal" solution should not deserve to be a preferred design solution if the "optimal" one cannot tolerate the small perturbations or the slight variations of design variables and optimized parameters [6]-[8]. This paper proposed a PSO based methodology in order to search the robust optimization in the solution space. In the proposed PSO, a four-quadrant-longest-distance method was developed to effectively evaluate the expected fitness value so as to determine the robust performance of the optimal solutions. The feasibility of the proposed PSO was demonstrated by the application to the mathematical robust optimization problem.

II. PROPOSED ALGORITHM

After define the search space and the swarm population size, generate the particles' locations and velocities randomly, the process of the proposed PSO is as follows:

A. Refresh Candidate Pool and Update Position (Step 1)

The position of a particle is determined according to the current location and velocity, then the object function is calculated for each updated particle, the particle with best objective function value is defined as leading candidate (P_{lc}). Fill the candidate pool with the whole particles, here, the candidate means the particles that will be used in the expected fitness evaluation.

B. Evaluate Expected Fitness (Step 2)

The fitness is used to measure the robust performance of a optimal solution. As mentioned in [6] and [7], the conventional fitness evaluation is too much computational burden to be viable for many engineering application. To overcome this issue, a four-quadrant-longest-distance method is introduced, i.e., the 2D space is divided into four-quadrant by using the leading candidate as the origin point. Next, in each quadrant, the particles are arranged in ascending order according to how far they are from the leading candidate. Simplified explanation on the candidate selection process is illustrated in Fig. 1. Based on the selection criterion that "the long distance the particle from P_{lc} the low objective function value the particle has," we can find in Fig. 1, P_3 is the criterion abiding particle with longest-distance, so we select P_3 as the common candidate. Therefore, there are at most four common candidates in four-quadrant that will be used in the expected fitness evaluation defined in (1).

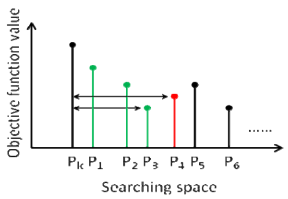

Fig. 1. A schematic illustration of candidate selection

$$f_{\exp}(x,y) = [f_{norm}(x,y)]^{\alpha} \cdot \left[\sum_{i=1}^{N} \left(\frac{1}{N} \sqrt{\frac{(x-x_i)^2 + (y-y_i)^2}{-\log(f_{norm}(x_i,y_i))}} \right) \right]^{\beta} \quad (1)$$

where N is the number of common candidates, x and y indicate the position of leading candidate, x_i and y_i describe the position of the ith common candidate, $f_{norm}(\cdot)$ is the normalized objective function of the optimal problem. α and β are weight parameters, large α corresponds to heavy reliance on the global (or local) optimal solution, while a large β gives heavy reliance on the robust solution. Therefore, we developed a fitness evaluation function that balances the contribution of both optimal and robust according to the application of the robust optimization.

C. Update Candidate Pool (Step 3)

Remove the criterion abiding particle and leading candidate, i.e., the P_1, P_2, P_3 and P_{lc} in Fig. 1, from the candidate pool. Steps (2-3) is repeated until the number of the candidate in candidate pool is less than 2.

D. Update pbests and gbests and Velocity (Step 4)

If the current fitness is better than the old individual best value, the *pbest* is replaced by the current position. The *gbest* is replaced by the best *gbest* among the swarm. The velocity of *j*th particle is updated based on (2) which is described clearly in [3] and [4]. Steps (1-4) are repeated until all particles are gathered around the *gbest*, or a maximum iteration is encountered.

$$\begin{aligned} Vx_j^{k+1} &= \omega\cdot Vx_j^k + C_p\cdot\varphi_p\cdot(pbestx_j^k - Px_j^k) + C_g\cdot\varphi_g\cdot(gbestx_j^k - Px_j^k) \\ Vy_j^{k+1} &= \omega\cdot Vy_j^k + C_p\cdot\varphi_p\cdot(pbesty_j^k - Py_j^k) + C_g\cdot\varphi_g\cdot(gbesty_j^k - Py_j^k) \end{aligned} \quad (2)$$

III. Numerical Test and Result

A 2D test mathematical function is used to verify the performance of the proposed algorithm formulated as,

$$f(x,y) = \sum_{k=1}^{M}\left(b_k \cdot e^{[(x-x_{pk})^2+(y-y_{pk})^2]/(-2\cdot a_k^2)}\right) \quad (3)$$

where M is the total number of peaks. a_k and b_k are, respectively, the width and amplitude of the kth peak. x_{pk} and y_{pk} are the position of the kth peak. x and y are the decision variables. The test function is generated according to [6] and the mathematical expression is formulated as

$$f(x,y) = 0.7e^{[(x-1)^2+(y-1)^2]/(-0.8)} + 0.75e^{[(x-1)^2+(y-3)^2]/(-3.2)} + e^{[(x-3)^2+(y-1)^2]/(-2)} + 1.2e^{[(x-3)^2+(y-4)^2]/(-0.32)} + e^{[(x-5)^2+(y-2)^2]/(-0.72)} \quad (4)$$

the search space is $0 \le x, y \le 5$. Fig. 2 shows the shape of the test function, where total number of local optimal solution is 5 at positions (1,1), (1,3), (3,1), (3,4) and (5,2). The global optimal solution lies in position (3,4), the robust optimal solution is located at position (3,1).

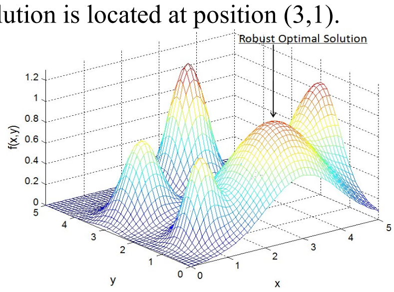

Fig. 2. Test mathematical function

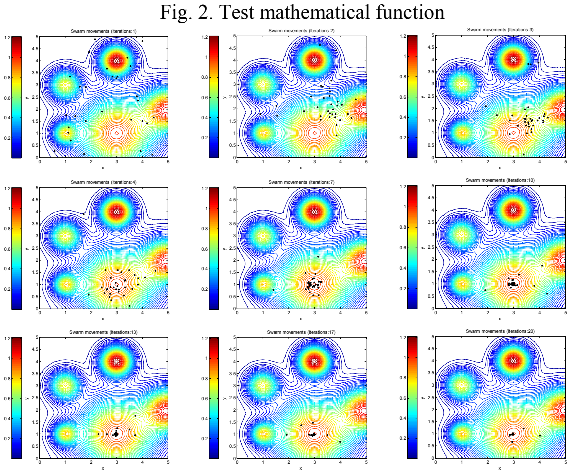

Fig. 3. Optimization result (α=1,β=1)

The basic conditions to execute the proposed PSO are defined as: number of iterations and particles are 20 and 36, ω is 0.6, C_p and C_g are 1.5. The optimization processes shown in Fig. 3 indicate that the proposed PSO can find robust optimal solution with a fast convergence rate. To observe effect of the proposed expected fitness evaluation method, we apply the proposed PSO by setting β value as 0 to the same test function and execution conditions. The results of the simulation is depicted in Fig. 4. The solutions converged to global optimal due to the absence of proposed robust performance evaluation of the optimal solutions.

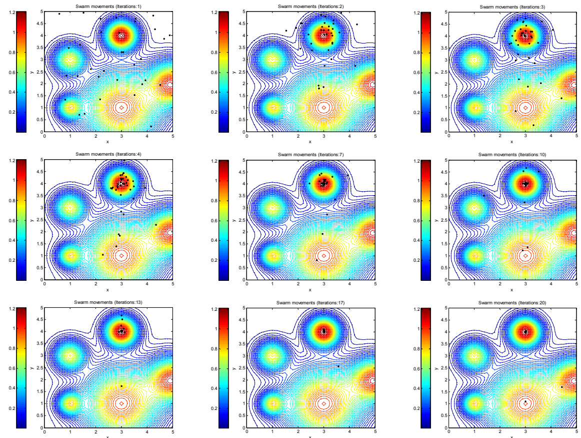

Fig. 4. Optimization result (α=1,β=0)

IV. Conclusion

This paper proposed a PSO algorithm with novel four-quadrant-longest-distance expected fitness evaluation for robust optimization problems. Through the application to numerical function, it was revealed that the proposed PSO algorithm is promising for robust optimization problems. The electromagnetic engineering application of the proposed algorithm will be demonstrated in the full paper.

V. References

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